

“No Fuss” Factoring For Second-Degree Polynomials

Factoring involves a certain amount of trial and error, which can become frustrating, especially when the leading coefficient is not 1. You might want to try a rather neat scheme that will greatly reduce the number of candidates.

We'll demonstrate the method for the polynomial

$$4x^2 + 11x + 6 \tag{1}$$

Using the leading coefficient of 4 we write the pair of incomplete factors

$$(4x \quad)(4x \quad) \tag{2}$$

Next, multiply the coefficient of x^2 and the constant term in (1) to produce $4 \cdot 6 = 24$. Now find two integers whose product is 24 and whose sum is 11, the coefficient of the middle term of (1). It's clear that 8 and 3 will do nicely, so we write

$$(4x+8)(4x+3) \tag{3}$$

Finally, within each parenthesis in (3) discard any common divisor. Thus $(4x+8)$ reduces to $(x+2)$ and we write

$$(x + 2)(4x + 3) \tag{4}$$

which is the factorization of $4x^2 + 11x + 6$.

Will the method always work? Yes—if you first remove all common factors in the original polynomial. That is, you must first write

$$6x^2 + 15x + 6 = 3(2x^2 + 5x + 2)$$

and apply the method to the polynomial $2x^2 + 5x + 2$.

(For a proof that the method works, see M. A. Autrie and J. D. Austin, “A Novel Way to Factor Quadratic Polynomials.”, *The Mathematics Teacher* 72 no. 2[1979].)

We'll use the polynomial $2x^2 - x - 6$ of Example 7 to demonstrate the method when some of the coefficients are negative.

Factoring $ax^2 + bx + c$	Example: $6x^2 + 7x - 3$
Step 1. Using the leading coefficient of a we write the pair of incomplete factors	Step 1. The lead coefficient is 6, so we write $(6x \quad)(6x \quad)$
Step 2. Multiply a and c , the coefficients of x^2 and the constant term.	Step 2. $a \cdot c = (6)(-3) = -18$
Step 3. Find integers whose product is $a \cdot c$ and the constant term	Step 3. Two integers whose product is -18 and whose sum is 7 are 9 and -2 . Then we write $(6x + 9)(6x - 2)$
Step 4. Discard any common factor within each parenthesis in Step 3. The result is the desired factorization.	Step 4. Reducing $(6x + 9)$ to $(2x + 3)$ and $(6x - 2)$ to $(3x - 1)$ we have $6x^2 + 7x - 3 = (2x+3)(3x-1)$